An integrated model and solution algorithms for passenger, cargo, and combi flight scheduling

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Abstract

In this research, we develop an integrated scheduling model that combines passenger, cargo and combi flight scheduling. We employ network flow techniques to construct the model which is formulated as an integer multiple commodity network flow problem that is characterized as NP-hard. A family of heuristics, based on Lagrangian relaxation, a sub-gradient method, heuristics for the upper bound solution, and a flow decomposition algorithm, is developed to solve the model. The test results, mainly using data from a major Taiwan airline's operations, show the good performance of the model and the solution algorithms.

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1. Introduction

In addition to the growing passenger demand, the demand for air cargo has increased drastically in Taiwan. According to the Taiwan Taoyuan International Airport (a major airport in Taiwan) statistics, the air cargo growth rate in Taiwan was about 10.17\% on average in 1991–2005. In such an environment, most passenger airlines have also been operating concurrent air cargo flights and here introduced combi (or combined) flights into their operations.

Unlike strictly passenger or cargo flights, these combi flights combine the transport of passengers and cargo on one aircraft at the same time. Both passenger and cargo transportation services are provided during regular operations. In particular, if an OD pair's passenger and cargo demands are both satisfied within a certain limit, then the use of combi flights will be of greater benefit than only passenger/cargo flights. That is, if the carrier can use combi flights to effectively handle passenger–cargo relationships, then both aircraft usage and operating performance could be improved. In terms of system optimization, it is important for carriers to...
consider the interrelationship between passenger, cargo and combi flight sources during fleet routing and flight scheduling.

Currently, most airlines in Taiwan use a trial-and-error process scheduling passenger, cargo and combi flights separately in the same season. The process is outlined as follows. The planners first determine the passenger and the combi timetables according to the projected passenger demand, the market share, and the time slots available at various airports. The feasibility checks are then made, especially for the fleet route, fleet size and fleet availability, as well as the related costs/revenue, the crew scheduling constraints, and the maintenance arrangements. After the passenger and combi timetables are made, the cargo timetable is then planned in accordance with the passenger and combi timetables as well as the projected cargo demand and other related constraints. Improvements may act as feedback to further revise the combi timetable. This process is iterated manually until a desirable timetable is obtained.

As the aforementioned discussions, the practical values of developing an integrated model that combines passenger, cargo and combi flight schedules could be considered on the following points:

1. The current flight scheduling process will result in the incapability of effectively handling the passenger, cargo and combi flight sources. To remedy this practical issue, the integrated model, with optimization from a systemic perspective, directly and systematically manages these flight sources together in the scheduling process.
2. As connected the passenger and cargo service networks, the integrated model is capable of effectively managing the aforementioned passenger–cargo relationships between the two networks, to improve airlines’ operating performance.
3. Using the integrated model can speed up the flight scheduling process, which also enhances cooperation among different processes, such as the aircraft maintenance and the crew scheduling processes.
4. As will be seen, the integrated model is designed as a systematic computerized tool, so that the passenger, combi, and cargo flight schedules can be simultaneously obtained and represented. It will be more practical and convenient for airline staffs to perform the scheduling activities.

In this research, on the basis of the carrier’s perspective, we develop an integrated scheduling model that combines passenger, cargo, and combi flight schedules. The objective is to maximize the operating profit, subject to the related operating constraints. This model is capable of directly managing the interrelationship between the passenger, the cargo and the combi flight sources and is expected to be a useful planning tool for determining their suitable fleet routes and timetables in short-term operations.

We employ a network flow technique to construct the model. It includes multiple fleet-flow, passenger-flow, and cargo-flow networks. The model is formulated as an integer multiple commodity network flow problem that is characterized as NP-hard. Since the problem size is expected to be huge, the model is more difficult to solve than traditional passenger/cargo flight scheduling problems. To more efficiently solve the model given practical problems, we refer to recent efforts made to solve similar types of network flow problems (Yan and Young, 1996; Yan and Tseng, 2002). A family of heuristics is developed based on Lagrangian relaxation, a sub-gradient method, two heuristics for the upper bound solution and a flow decomposition algorithm. The development of an effective model, as well as an efficient solution algorithm, together are the focus of this study.

It should be mentioned that we only consider the combi flights to be served by the combi aircraft. Although the passenger aircraft can also stow belly cargos under the main deck, the space supply for belly cargos usually needs to be dependent on the volume/weight of passenger baggage. If the belly cargo is considered in the scheduling, then the belly cargo (or passenger baggage) capacity in the passenger aircraft has to be a decision variable in the model. In addition, the combi aircraft sometimes allows the seats to be added or removed, implying that the passenger and cargo capacities may be variable, although an average passenger or cargo capacity can be practically used in the model. That is, the passenger or cargo capacity in the combi aircraft will also be a decision variable in the model. Consequently, the problem with either of the two issues will become a complicated non-linear integer program, which will be significantly more difficult to solve than the current one. Therefore, to facilitate problem solving, the incorporations of the two issues are left for directions of future research.
Currently for this Taiwan airline, airport selection is typically part of long-term planning. It is usually performed before fleet routing and timetable setting. Thus, the selection at airport is not considered here. However, this model is extendable for such an application, when airport selection, fleet routing and timetable setting must be integrated together in short-term operations. In addition, the scope of this research is confined to pure fleet routing and flight scheduling. Although the scheduling process is closely related to the aircraft maintenance and the crew scheduling processes, these processes are usually separated, to facilitate problem solving (Teodorovic, 1988). Recently, Clarke et al. (1996) tried to develop a fleet assignment model that considered both maintenance and crew scheduling. However, according to the studied Taiwan airline, in practice, maintenance and crew constraints are rather flexible, due to the use of stand-by crews and a progressive maintenance policy. These activities are always planned after the fleet routes and flights schedules have been determined. To reduce the problem complexity, as in Yan and Young (1996) and Yan and Tseng (2002), we thus exclude these constraints in the modeling. The incorporation of these constraints into fleet/flight scheduling could be a topic of future research.

The rest of this paper is organized as follows: Section 2 reviews the related literatures. Section 3 introduces the proposed model. Section 4 develops the solution algorithms for solving the model. Section 5 describes the numerical tests. Finally, we conclude in Section 6.

2. Literature review

Past research on fleet routing and flight scheduling has naturally been for passenger and cargo transportation. In particular, fleet routing and flight scheduling for passenger transportation has been performed by many researchers, for example, by Levin (1969, 1971), Simpson (1969), Abara (1989), Dobson and Lederer (1993), Hane et al. (1995), Clarke et al. (1996), Yan and Young (1996), Desaulniers et al. (1997), Yan and Tseng (2002), Barnhart et al. (2002), and Lohatepanont and Barnhart (2004). These models have been formulated as integer linear programs, mixed integer programs or multicommodity network flow problems. The objective has been to minimize a carrier’s operating cost or to maximize the system profit. The problems usually have been solved using exact solution methods or heuristics, such as the simplex method, the branch-and-bound technique, the cutting plane method, the Lagrangian relaxation-based algorithm, the column and row generation technique and other heuristics.

Compared with passenger transportation, there has been little research on fleet routing and flight scheduling for cargo transportation, for example, Antes et al. (1998), Lin and Chen (2003) and Yan et al. (2006). In particular, to handle multiple on-time cargo demands in reality, Yan et al. (2006) employed network flow techniques to construct their model, which includes multiple cargo-flow and fleet-flow networks. Apart from fleet routing and flight scheduling, air cargo transportation problems have been studied by a variety of topics. For example, service network and frequency planning (Current et al., 1986; Current, 1988; Lin, 2001; Lin et al., 2003), the characteristics of air express carriers (Chan and Ponder, 1979; Chestler, 1985), hub/airport location selection (Aykin, 1995; JaiIket al., 1996; Sohn and Park, 1997; Sue, 1998; Nero, 1999), air freight strategies as well as competition and configuration analyses (Lederer, 1993; Prokop, 2002; Zhang and Zhang, 2002; and Zhang, 2003), and short-term booking of air cargo space (Chew et al., 2006). Although some characteristics and strategies about the cooperation of passenger and cargo flights/airlines have been discussed, they have a different focus than this research, and, therefore, do not provide a suitable fleet route and flight schedule for short-term operations.

From the literature review, the main contributions of our research to previous studies are addressed as follows:

1. Past research on fleet routing and flight scheduling handled passenger and cargo transportation problems separately. Unlike previous studies, we develop an integrated model that considers the interrelationships between passenger, cargo, and combi flights to assist airlines in solving their fleet routing and flight scheduling problems.

2. Due to the fact that our model is more difficult to solve than traditional passenger/cargo flight scheduling problem, a family of heuristics is developed in this research. The development of the solution algorithm for solving the integrated passenger, cargo, and combi flight scheduling problem was not discussed in past research.
3. The model

A multiple time–space network technique is applied to construct an integrated scheduling model. This model demands the optimal management of aircraft, passenger, and cargo movements within the network. The major elements in the modeling, including the fleet-flow time–space networks, the passenger-flow time–space networks, the cargo-flow time–space networks, and the mathematical formulation, are described as follows.

3.1. The fleet-flow time–space networks

We adopt several time–space networks to formulate the multi-type fleet routing and flight scheduling problem. As shown in Fig. 1, each network indicates one specific type of potential fleet movement (i.e., passenger-
fleet, cargo-fleet, or combi-fleet) within a certain time period (one week in this study) and a specific location. The vertical axis represents the time duration, while the horizontal axis indicates the airport location. A node stands for an airport at a specific time, while an arc designates an activity for an airplane. The arc flows express the flow of airplanes in the networks. Three types of arcs are defined.

3.1.1. Flight leg arc
A flight leg arc, marked by (1) in Fig. 1, represents a flight connecting two different airports. All possible flight legs between two available airports are installed into the network within a reasonable block of time, assuming that time slots at the corresponding airports are available. Each flight leg arc contains the following information: the departure time, the departure airport, the arrival time, the arrival airport, and the operating cost. The time block for a flight leg includes: the investigation time prior to departure, as well as fuelling, passengers planning/deplaning (passenger-fleet), cargo loading/unloading (cargo-fleet), or both (combi-fleet) times, and of course flight time in the air. In general, the time is calculated as from the time when the airplane begins to prepare for a particular flight leg to the time when this flight leg is finished. The arc cost is the overall operating cost for a particular flight. The arc flow’s upper bound is set to be one, showing that at most one airplane can serve this flight leg. The arc flow’s lower bound is set to be zero, implying that no airplane serves this flight leg. In addition, the departure interval at the same airport is designed to be adjustable, so as to be sensitive to a carrier’s operating requirements.

3.1.2. Ground arc
The ground arc, marked by (2) in Fig. 1, indicates the holding or the overnight stay of an aircraft at an airport in a time window. The arc cost is the cost for holding an aircraft at an airport in the corresponding time window. It includes the airport tax, the airport holding (or overnight stay) fee, the gate use charge and any other related costs. The arc flow’s upper bound is set to be the apron capacity (or infinity, if the capacity is large), indicating the maximum number of airplanes that can be held at this airport during a specific time window. The arc flow’s lower bound is set to be zero, implying that no airplane is held at this airport in this time window.

3.1.3. Cycle arc
A cycle arc, marked by (3) in Fig. 1, shows the continuity between two consecutive planning periods. It connects the end of one period to the beginning of the next period, for each airport. The arc cost is the cost of holding an airplane overnight. The arc flow’s upper bound and lower bound are the same as those of the ground arcs.

3.2. The passenger-flow time–space networks
The time–space network technique is also applied to indicate passenger movements corresponding to certain times and locations. As shown in Fig. 2, each passenger-flow time–space network represents a specific OD pair from the origin–destination table (known as the OD table). Such networks are designed to symmetrically correspond to the passenger-fleet-flow and combi-fleet-flow time–space networks, so as to facilitate problem solving. The horizontal and vertical axes are defined as the same as those in the fleet-flow networks. Similarly, a node represents an airport at a specific time, while an arc designates an activity showing passenger movement. There are three types of arcs.

3.2.1. Passenger delivery arc
The passenger delivery arc as marked by (1) in Fig. 2 represents the transportation of passengers from one airport to another on a flight leg. The transportation time is the same as the corresponding time block for the associated flight leg in the passenger-fleet-flow and combi-fleet-flow time–space networks. The arc cost is a variable cost for serving each passenger (e.g. the catering cost). The arc flow’s upper bound is the airplane’s passenger capacity (particularly for the passenger-fleet). The arc flow’s lower bound is zero, indicating that no passengers from the corresponding OD are delivered on the associated flight leg.
3.2.2. Passenger holding arc

The passenger holding arc as marked by (2) in Fig. 2 denotes passengers staying at an airport in a time window. The arc cost is a waiting (or penalty) cost for the associated time window. However, if the arc just so happens to connect either the departure or the arrival station of this network’s corresponding OD pair, then the arc cost is set to be zero, because no passenger would, practically, make an unnecessary stay at the departure or arrival airports. This cost is usually not considered by the carrier. Nevertheless, the arc cost is adjustable in each application. The arc flow's upper bound is the station’s passenger service capacity (or infinity if the capacity is relatively large). This is the maximum number of passengers that can be accommodated at this airport in this time window. The arc flow’s lower bound is zero, meaning that no passenger from the corresponding OD pair stays at the airport during this time window.

(1) Passenger delivery arc  (2) Passenger holding arc  (3) Passenger demand arc

Fig. 2. Passenger-flow time–space networks.
3.2.3. Passenger demand arc

The passenger demand arc as marked by (3) in Fig. 2 shows the actual service demand for an OD pair. It connects the arrival station to the departure station of the corresponding network OD pair. The arc cost is the negative value of the average ticket fare (i.e. revenue). The arc flow’s upper bound is the projected demand for this OD pair for the given time interval. The aim of the model is to maximize carrier profit, which implies that not all the passengers for this OD pair will necessarily be served. The arc flow’s lower bound is set to be zero, indicating that none of the OD pair’s passengers are served in the network. The trip demand for a specific OD pair can be flexibly divided into several demand arcs according to the actual demand distribution, the market characteristics, or carrier own considerations. For example, the arcs for an OD pair could be denser (e.g. one arc per hour), with more commuter trips during peak hours. On the other hand, the arcs could be sparsely distributed in the network (e.g. one arc every 3 h) if passengers are less sensitive to time, for example, for leisure trips. The time intervals for the demand arcs are adjustable. If the model results are expected to affect the original demand, one can change the inputs and rerun the model until satisfactory results are acquired.

3.3. The cargo-flow time–space networks

The cargo-flow time–space networks shown in Fig. 3 indicate cargo movements corresponding to certain times and locations. According to the time sensitivity cargos are divided into three time types, one day, four days, and one week. Therefore, unlike the fleet-flow/the passenger-flow time–space networks, each cargo-flow time–space network represents a specific OD–time-pair (e.g., OD pair 1-2 within one day). Note that the lengths of time are adjustable, according to actual cargo timeliness. For example, the time of the corresponding cargo-flow time–space network could be shorter for express deliveries, or longer for a less time sensitive cargo. These networks are designed to symmetrically correspond to the cargo-fleet-flow and the combi-fleet-flow time–space networks, so as to facilitate problem solving. The horizontal and vertical axes are defined as the same as those in the fleet-flow time–space networks. Also, a node represents an airport at a specific time, while an arc designates an activity showing cargo movement. Three types of arcs are described.

3.3.1. Cargo delivery arc

A cargo delivery arc, marked by (1) in Fig. 3, represents the transportation of cargo from one station to another on a flight leg. The transportation time is the same as the corresponding time block for the associated flight leg in the cargo-fleet-flow and the combi-fleet-flow time–space networks. The arc cost is a variable cost for handling the cargo, per unit weight, on this flight, and is, in general, very small compared to the flight cost. The arc flow’s upper bound is the aircraft’s cargo capacity (particularly for the cargo-fleet), the weight capacity for the studied airline. The arc flow’s lower bound is set to be zero, meaning that no cargo from the corresponding OD is delivered on the associated flight leg.

3.3.2. Cargo holding arc

A cargo holding arc, marked by (2) in Fig. 3, indicates that the cargos are held at an airport in a time window. The arc cost for this time window is a holding (or penalty) cost. In practice, the holding of a cargo before or after delivery is usually not decided by the airline. Therefore, similar to the passenger holding arc, if the arc just happens to connect either the departure or the arrival station of this network’s corresponding OD–time-pair, the arc cost is then zero. Certainly, it is adjustable. A suitable holding cost for special cases can be imposed. The arc flow’s upper bound is the station’s cargo service capacity (or infinity, if the capacity is relatively large), meaning that the maximum amount of cargo (in weight units appropriate for the studied airline) can be accommodated at this airport. The arc flow’s lower bound is set to be zero, indicating that no cargo from the corresponding OD–time-pair is held at the airport during the time window.

3.3.3. Cargo demand arc

A cargo demand arc, see (3) in Fig. 3, shows the service demand for the OD–time-pair that would actually be served in the network. It connects the arrival station to the departure station of the corresponding network
OD–time-pair. The time interval for the demand arcs (i.e. the arc density) is set to be the time length of the corresponding cargo-flow time–space network (i.e. one day, four days, or one week). The arc cost is the negative value of the cargo per unit weight delivered. Note that the cargo fare structure that an airline uses to charge a forwarder or a shipper is, in general, in a decreasing ladder form (i.e. a concave function in terms of accumulation of cargo). However, the cargo amount transport on a flight is far larger than the amount charged an individual forwarder or shipper. Consequently, in the flight scheduling, an average cargo fare per unit weight is usually used. The arc flow’s upper bound is the projected demand for this OD–time-pair. The aim is to maximize carrier profit, meaning that in the model for this OD–time-pair not all cargo will necessarily be served in the model. The arc flow’s lower bound is set to be zero, implying that none of the OD–time-pair’s cargos are served in the network.
3.4. The model formulation

In addition to the three aforementioned major elements, there are several operating constraints that need to be considered, including specifically the number of available airplanes in each fleet, the quota for each airport/airport pair, and the airplane’s capacity, respectively. As well, the same flight leg in the passenger-fleet-flow and the combi-fleet-flow networks can be served at most once. Similarly, the same flight leg in the cargo-fleet-flow and the combi-fleet-flow networks can be served at most once. Note that according to current Taiwan airline regulation, combi flights are usually incorporated into passenger flights, for determining the quota for each airport/airport pair.

Given the fleet-flow, the passenger-flow, and the cargo-flow time–space networks introduced above, as well as the operating constraints, we can now formulate the model as a mixed integer problem. The objective of this model is to simultaneously “flow” the airplanes, the passengers, and the cargos in all networks, at a minimum cost. Since the revenues from the passenger-flow and the cargo-flow networks are included in the form of negative costs, this objective is equivalent to the maximization of profit. In order to facilitate problem solving, we set all the arc flows in each passenger-flow and cargo-flow network to be real variables, since such simplification does not have a significant effect on the results in the planning stage (Yan and Chen, 2002).

The model formulation is shown as follows:

Model (A):

Objective function:

\[
\text{Min} \sum_{m \in M} \sum_{ij \in A_m} C_{ij} X_{ij}^m + \sum_{l \in L} \sum_{ij \in D_l} O_{ij}^l P_{ij}^l + \sum_{n \in N} \sum_{ij \in B_n} T_{ij}^n Y_{ij}^n + \sum_{m \in M} \sum_{ij \in BF_m} Y_{ij}^m (V_i + V_j). \tag{1}
\]

Model (A) is formulated as a mixed integer multiple commodity network flow problem, in which the objective function (1) is to minimize the system cost, which is equivalent to the maximization of profit, where \(X_{ij}^m\), \(P_{ij}^l\), and \(Y_{ij}^n\) are the arc \((i,j)\) flow in the \(m\)th fleet-flow, the \(l\)th passenger-flow, and the \(n\)th cargo-flow networks, respectively; \(C_{ij}^m\), \(O_{ij}^l\), and \(T_{ij}^n\) are the arc \((i,j)\) cost in the \(m\)th fleet-flow, the \(l\)th passenger-flow, and the \(n\)th cargo-flow networks, respectively; \(V_i\) is a variable cost at station \(i\) for handling cargo per unit weight, including the loading and unloading cargo; \(m\), \(M\) are the \(m\)th type of fleet (i.e. the \(m\)th fleet-flow network) and the set of all fleets. In particular, \(m = 1, 2, 3\) represent the passenger-fleet-flow, the combi-fleet-flow, and the cargo-fleet-flow networks, respectively; \(l\), \(L\) are the \(l\)th OD pair (corresponding to the \(l\)th passenger-flow network) and the set of all ODs; \(n\), \(N\) are the \(n\)th OD–time pair (i.e. the \(n\)th cargo-flow network) and the set of all OD–time pairs; \(A_m\), \(D_l\), \(B_n\) are the set of all arcs in the \(m\)th fleet-flow, the \(l\)th passenger-flow, and the \(n\)th cargo-flow networks, respectively; and \(BF_m\) is the set of all cargo demand arcs in the \(n\)th cargo-flow network.

Flow conservation constraints:

\[
\sum_{j \in NF_m} X_{ij}^m - \sum_{k \in NF_m} X_{ki}^m = 0 \quad \forall i \in NF_m, \quad \forall m \in M, \tag{2}
\]

\[
\sum_{j \in NG_l} P_{ij}^l - \sum_{k \in NG_l} P_{kj}^l = 0 \quad \forall i \in NG_l, \quad \forall l \in L, \tag{3}
\]

\[
\sum_{j \in NF_n} Y_{ij}^n - \sum_{k \in NF_n} Y_{kj}^n = 0 \quad \forall i \in NF_n, \quad \forall n \in N, \tag{4}
\]

constraints (2)–(4) ensure flow conservation at every node in each fleet-flow, passenger-flow or cargo-flow network, where \(NF_m\), \(NG_l\), \(NF_n\) are the set of all nodes in the \(m\)th fleet-flow, the \(l\)th passenger-flow, and the \(n\)th cargo-flow networks, respectively.

Available airplane constraint:

\[
\sum_{ij \in CF_m} X_{ij}^m \leq AF_m \quad \forall m \in M, \tag{5}
\]

constraint (5) indicates that the number of airplanes used in each fleet-flow network should not exceed the number of available airplanes for that fleet, where \(AF_m\) is the number of available airplanes in the \(m\)th fleet-flow network and \(CF_m\) is the set of all cycle arcs in the \(m\)th fleet-flow network.
Flight leg service constraints:
\[
\sum_{m \in \{1, 2\}} X^m_{ij} \leq 1 \quad \forall i, j \in FF_1 \cap FF_2,
\]
\[
\sum_{m \in \{2, 3\}} X^m_{ij} \leq 1 \quad \forall i, j \in FF_2 \cap FF_3,
\]
constraints (6) and (7) denotes that the same flight leg, in the passenger-fleet-flow and the combi-fleet-flow networks as well as in the combi-fleet-flow and the cargo-fleet-flow networks, is served at most once, respectively, where \( FF_m \) is the set of all flight leg arcs in the \( m \)th fleet-flow network.

Airport pair flight quota constraints:
\[
\sum_{m \in \{1, 2\}} \sum_{ij \in SE^{ab}} X^m_{ij} \leq QA^{ab} \quad \forall ab \in SA,
\]
\[
\sum_{ij \in SF^{ab}} X^3_{ij} \leq QB^{ab} \quad \forall ab \in SB,
\]
constraints (8) and (9), respectively, ensure that the sum of all flights for each airport pair does not exceed the approved passenger flight and cargo flight quotas, where \( SE^{ab} \) and \( SF^{ab} \) are the set of all flight leg arcs that connect the \( a \)th to the \( b \)th stations in both the passenger- and combi-fleet-flow networks, and the cargo-fleet-flow network; \( QA^{ab} \) and \( QB^{ab} \) are the approved passenger flight and cargo flight quotas that connect the \( a \)th to the \( b \)th stations; and \( SA \) and \( SB \) are the set of airport pairs with approved passenger flight and cargo flight quotas.

Airplane capacity constraints:
\[
\sum_{m \in \{1, 2\}} \sum_{ij \in SG^a} X^m_{ij} \leq QC^a \quad \forall a \in SC,
\]
\[
\sum_{ij \in SD^a} X^3_{ij} \leq QD^a \quad \forall a \in SD,
\]
constraints (10) and (11), respectively, ensure that the sum of all flights at each station does not exceed its approved passenger flight and cargo flight quotas, where \( SG^a \) and \( SD^a \) are the set of all flight leg arcs associated with the \( a \)th station in both the passenger- and combi-fleet-flow networks, and the cargo-fleet-flow network; \( QC^a \) and \( QD^a \) are the approved passenger flight and cargo flight quotas at the \( a \)th station; and \( SC \), \( SD \) are the set of all stations in both the passenger- and combi-fleet-flow networks, and the cargo-fleet-flow network.

Airplane capacity constraints:
\[
\sum_{l \in L} P^l_{ij} \leq (\alpha_{ij} X^1_{ij} + \beta_{ij} X^2_{ij}) \quad \forall i, j \in FF_1 \cap FF_2,
\]
\[
\sum_{m \in N} Y^m_{ij} \leq (\gamma_{ij} X^1_{ij} + \delta_{ij} X^3_{ij}) \quad \forall i, j \in FF_2 \cap FF_3,
\]
constraints (12) and (13), respectively, keep the passenger and cargo delivery rates within the airplane’s carrying capacity, where \( \alpha_{ij}, \beta_{ij} \) are the aircraft’s passenger capacity in the passenger-fleet-flow and the combi-fleet-flow networks (a planning load factor could be used in the planning stage) and \( \gamma_{ij}, \delta_{ij} \) are the aircraft’s cargo capacity in the combi-fleet-flow and the cargo-fleet-flow networks (a planning load factor could be used in the planning stage).

Arc flow bound constraints and airplane flow integrality constraint:
\[
0 \leq X^m_{ij} \leq U^m_{ij} \quad \forall i, j \in A_m, \quad \forall m \in M,
\]
\[
0 \leq P^l_{ij} \leq U^l_{ij} \quad \forall i, j \in D_l, \quad \forall l \in L,
\]
\[
0 \leq Y^m_{ij} \leq U^m_{ij} \quad \forall i, j \in B_n, \quad \forall n \in N,
\]
\[
X^m_{ij} \in \text{Integer} \quad \forall i, j \in A_m, \quad \forall m \in M,
\]
constraints (14)–(16) hold that all the arc flows are within their upper and lower bounds. Constraint (17) ensures the integrality of the airplane flows, where \( U^m_{ij} \), \( U^l_{ij} \), \( U^m_{ij} \) are the arc(\( i,j \)) flow’s upper bound in the \( m \)th fleet-flow, the \( l \)th passenger-flow, and the \( n \)th cargo-flow networks, respectively.
4. Solution algorithm

The model is formulated as a mixed integer program that is characterized as NP-hard. We adopt the Lagrangian relaxation technique, coupled with a sub-gradient method, to develop a family of heuristics to solve the problem. The solution processes of the heuristic methods are the same. We first relax the side constraints (constraints (5)–(13)) to construct a Lagrangian problem, and then solve it, to procure the lower bound of the optimal solution. Secondly, two heuristics, UP1 and UP2, are developed to solve for the upper bound of the optimal solution. Then, a sub-gradient method for revising the Lagaragian multipliers is utilized to iterate the lower and upper bounds, until an acceptable convergence result is reached, or until the number of iterations exceeds a preset number. For ease of writing, we first define the family of heuristics. The main differences are as follows:

1. UP1_Z: The upper bound of the optimal solution is found using UP1 and the initial Lagrangian multipliers are set to be zero.
2. UP2_Z: The upper bound of the optimal solution is found using UP2 and the initial Lagrangian multipliers are set to be zero.
3. UP1_O: The upper bound of the optimal solution is found using UP1 and the initial Lagrangian multipliers are set to be the extra volume of the violated constraints for the optimal solution of the linear program (by relaxing the integer constraints (17)).
4. UP2_O: The upper bound of the optimal solution is found using UP2 and the initial Lagrangian multipliers are set to be the extra volume of the violated constraints for the optimal solution of the linear program (by relaxing the integer constraints (17)).

4.1. UP1_Z

The major parts of UP1_Z, including the lower bound of the optimal solution, the upper bound of the optimal solution, the sub-gradient method and the solution process, are addressed as follows:

4.1.1. The lower bound of the optimal solution

The steps for searching for the lower bound are listed below (the formulations of Models (B), (C), (D), (E), (F), and (G) used in the steps are shown in the Appendix):

Step 1: Side constraints (5)–(13) are relaxed with the corresponding non-negative Lagrangian multiplier sets, \( \mu^5 \) to \( \mu^{13} \), and are added to the objective function of Model (A), resulting in Model (B). The optimal objective value for Model (B) becomes the lower bound of Model (A).

Step 2: Model (B) is decomposed, omitting the constant terms (e.g. \( -\mu S_m 4F_m \)), into five independent groups of networks, such as Model (C), the passenger-fleet-flow network, Model (D), the combi-fleet-flow network, Model (E), the cargo-fleet-flow network, Model (F), the passenger-flow networks, and Model (G), the cargo-flow networks.

Step 3: Models (C), (D), (E), (F), and (G) are pure network flow problems, and are also characterized as minimum cost network flow problems, which can be solved directly using the mathematical programming solver, CPLEX.

Step 4: The lower bound of the optimal solution is obtained by summing up all five network costs and the constant terms.

4.1.2. The upper bound of the optimal solution

UP1 is used to find an upper bound (a feasible solution). The searching process is outlined in Fig. 4, and the steps are listed below. For ease of introduction, we first define the symbols that are used in the heuristic as follows:

PFFN, CFFN, CAFFN: the passenger-fleet-flow network, the combi-fleet-flow network and the cargo-fleet-flow network.
FFNS, PFNS, CFNS: the fleet-flow networks, the passenger-flow networks and the cargo-flow networks. In particular, FFNS includes PFFN, CFFN and CAFFN.

PFN, CFN: a passenger-flow network and a cargo-flow network.

MFFNS, MPFNS, MCFNS: the modified fleet-flow networks, the modified passenger-flow networks and the modified cargo-flow networks.

$p_{\text{ff}}^a$, $p_{\text{ff}}^b$: the passenger-fleet flows in the $i$th iteration, where subscripts $a$ and $b$ indicate whether the flows are infeasible or feasible.

Fig. 4. Upper bound search process.
cff$^a_i, cff^b_i$: the combi-fleet flows in the $i$th iteration, where subscripts $a$ and $b$ indicate whether the flows are infeasible or feasible.

caff$^a_i, caff^b_i$: the cargo-fleet flows in the $i$th iteration, where subscripts $a$ and $b$ indicate whether the flows are infeasible or feasible.

ff$^a_i, ff^b_i$: the fleet flows in the $i$th iteration, where subscripts $a$ and $b$ denote whether the flows are infeasible or feasible. ff$^a_i$ includes pff$^a_i$, cff$^a_i$ and caff$^a_i$, while ff$^b_i$ includes pff$^b_i$, cff$^b_i$ and caff$^b_i$.

pff$^a_i, pff^b_i$: the passenger flows in the $i$th iteration, where subscripts $a$ and $b$ indicate whether the flows are infeasible or feasible.

cff$^a_i, cff^b_i$: the cargo flows in the $i$th iteration, where subscripts $a$ and $b$ indicate whether the flows are infeasible or feasible.

$\Delta$pff$^a_i, \Delta$pff$^b_i$: the increased passenger flows for fubs$_i$, where subscripts $a$ and $b$ indicate whether the increased flows do not or do assure that the passenger delivery volume is within the aircraft capacity.

$\Delta$cff$^a_i, \Delta$cff$^b_i$: the increased cargo flows for fubs$_i$, where subscripts $a$ and $b$ indicate whether the increased flows do not or do assure that the cargo delivery volume is within the aircraft capacity.

cf$^a_i, cf^b_i$: the feasible upper bound solution in the $i$th iteration, including ff$^a_i$, pf$^a_i$, and cb$^a_i$.

ov_fubs$_i$: the objective value of fubs$_i$.

nfpf$^a_{i+1}, ncfb^a_{i+1}$: the new feasible passenger and cargo flows in the $(i+1)$th iteration.

nfubs$_{i+1}$: the new feasible upper bound solution in the $(i+1)$th iteration.

ov_nfubs$_{i+1}$: the objective value of nfubs$_{i+1}$.

The steps of UP1 are listed below:

**Step 1:** Calculate the feasible and the cargo flows for the initial lower bound solution be pf$^a_1$ and cf$^a_1$, respectively. Solve ff$^b_1$ (including pff$^b_1$, cff$^b_1$ and caff$^b_1$), based on pf$^a_1$ and cf$^a_1$ as follows. First, the modified fleet-flow network (MFFNS) are constructed the same as FFNS (including the fleet size and other related constraints), except for the cost of each flight leg arc. In particular, if the flight leg arc is in PFFN/CAFFN, then the arc cost must include the original operating cost plus the sum of all profits obtained from the corresponding passenger/cargo delivery arc flows of pf$^a_1$/cf$^a_1$. If the flight leg arc is in CFN, then the arc cost includes the original operating cost plus the sum of all profits obtained from both the corresponding passenger and cargo delivery arc flows of pf$^a_1$/cf$^a_1$. Note that the sum of the profits is set to be, at most, the aircraft capacity. Now, solve MFFNS, using CPLEX, to find ff$^b_1$.

**Step 2:** Solve pf$^b_1$ based on pf$^a_1$ and cf$^a_1$. We construct the modified passenger-flow networks (MPFNS) which are the same as PFNS, except that the passenger delivery arc flows in MPFNS are restricted by the passenger loading constraint (12), based on pf$^b_1$ and cf$^b_1$. Then, solve MPFNS, using CPLEX, to find pf$^b_1$.

**Step 3:** Find cf$^b_1$ based on cff$^b_1$ and caff$^b_1$. We construct the modified cargo-flow networks (MCFNS) which are the same as CFNS, except that the cargo delivery arc flows in MCFNS are restricted by the cargo loading constraint (13), based on cff$^b_1$ and caff$^b_1$. Then, solve MCFNS to find cf$^b_1$ using CPLEX.

**Step 4:** Achieve an fubs$_1$, and its objective value, ov_fubs$_1$ by combining ff$^b_1$, pf$^b_1$, and cf$^b_1$. Set $i = 1$.

**Step 5:** Solve $\Delta$pff$^a_i$ based on fubs$_i$ to increase unserved passengers as follows. First, for every passenger delivery arc in each PFN, the flow upper bound is reset to be the capacity of the aircraft associated with the flight leg arc in pf$^b_i$ or cf$^b_i$, minus its arc flow in pf$^b_i$. For every passenger demand arc in each PFN, recalculate the residual demand as the flow upper bound, which is equal to the projected passenger demand minus its arc flow in pf$^b_i$. Then, solve PFNS to find $\Delta$pff$^a_i$ using CPLEX, based on the new flow upper bounds of the passenger delivery and the passenger demand arcs (all other parameters remain the same). Finally, use the same way to update the flow upper bound for every passenger delivery and passenger demand arcs.

**Step 6:** Add up pf$^b_i$ and $\Delta$pff$^a_i$ to form pf$^b_{i+1}$, which, together with pf$^b_i$, usually violates the passenger loading constraint (12).

**Step 7:** Using the same technique as in Step 5, solve $\Delta$cff$^a_i$ based on fubs$_i$ to increase the unserved cargos.
Step 8: Add up $cf^b_i$ and $\Delta cf_{\text{ubs}}^b$ to form $cf^a_{i+1}$, which, along with $cfl^b_i$ and $caff^b_i$, usually violates the cargo loading constraint (13).

Step 9: Referring to Step 1, solve $pf^b_{i+1}$ (including $pf^b_{i+1}$, $cfl^b_i$, and $caff^b_i$) based on $pf^a_{i+1}$ and $cf^a_{i+1}$ obtained form Steps 6 and 8.

Step 10: Referring to Step 2, solve $pf^b_{i+1}$ based on $pf^b_{i+1}$ and $caff^b_i$.

Step 11: Referring to Step 3, solve $cf^b_{i+1}$ based on $cfl^b_{i+1}$ and $caff^b_i$.

Step 12: By combining $pf^b_{i+1}$, $pf^b_{i+1}$ and $caff^b_i$, we find the $fubs_{i+1}$ and its objective value, ov_fubs_{i+1}.

Step 13: If ov_fubs_{i+1} is better than ov_ubs, then set $i = i + 1$ and go to Step 5; else, go to Step 14.

Step 14: Solve $\Delta pf_{\text{ubs}}^b$ based on $fubs_{i+1}$ as follows. The method is similar to Step 5, except that the recalculation of the flow upper bound of every passenger delivery arc is different. To increase the passenger demand without violating constraint (12), we do not allow any flow to be augmented into the passenger delivery arcs when the corresponding flight leg arc flows in both $pf^b_{i+1}$ and $cfl^b_i$ are zero. In other words, for every passenger delivery arc, if the associated flight leg arcs in $pf^b_{i+1}$ and $cfl^b_i$ both equal zero, then its flow upper bound is set as zero. Using the same technique as in Step 5, we can solve $\Delta pf_{\text{ubs}}^b$.

Step 15: Add up $pf^b_{i+1}$ and $\Delta pf_{\text{ubs}}^b$ to form a new $pf^b_{i+1}$ (nfpf_{i+1}).

Step 16: Using the same technique as in Step 14, solve $\Delta cf_{\text{ubs}}^b$ based on $fubs_{i+1}$.

Step 17: Add up $cf^b_i$ and $\Delta cf_{\text{ubs}}^b$ to form a new $cf^b_{i+1}$ (nfcf_{i+1}).

Step 18: Find a new $f^b_{i+1}$ (nfpf_{i+1} and nfpf_{i+1}) based on nfpf_{i+1} and nfpf_{i+1}. To do this, we first fix the nfpf_{i+1} and nfpf_{i+1} variables in the objective function (1), constraints (3), (4), (12), (13), (15), and (16), and then solve the rest of Model (A). Finally, by combining nfpf_{i+1}, nfpf_{i+1}, and nfpf_{i+1}, we find a new feasible solution and its objective value, that is, nfpsb_{i+1} and ov_nfubs_{i+1}.

Step 19: If ov_nfubs_{i+1} is better than ov_ubs, then update fubs_{i+1} and ov_ubs, and go to Step 14; else, we find the final feasible solution, fubs_{i+1}.

4.1.3. The sub-gradient method and the solution process

Yan and Young’s (1996) sub-gradient method, for adjusting Lagrangian multipliers is applied in this research, so as to obtain good convergence in the iteration results. The steps of UPI_Z, a Lagrangian relaxation-based algorithm, are shown as follows:

Step 1: Set iteration $i = 0$ and the initial Lagrangian multiplier $\mu'$ to be 0.

Step 2: Use CPLEX to solve Models (C), (D), (E), (F), and (G) to get a lower bound, $Z^L(\mu')$. If the solution is feasible and also satisfies the complementary slackness condition, then we have found an optimal solution and can stop the solution process. Otherwise, update the lower bound, $Z^L$.

Step 3: Apply the UP1 to find an upper bound, $Z^U(\mu')$, and update the upper bound, $Z^U$.

Step 4: If the gap between the lower bound, $Z^L$, and the upper bound, $Z^U$, falls within a specified tolerance, $\theta$ (i.e. $|Z^U - Z^L|/Z^U| \leq \theta$), or the number of iterations reaches a preset limit, stop the algorithm.

Step 5: Adjust $\mu'$ to help improve the convergence by applying the sub-gradient method developed in Yan and Young (1996).

Step 6: Set $i = i + 1$. Go to Step 2.

4.2. UP2_Z, UP1_O, and UP2_O

4.2.1. UP2_Z

In this heuristic, we apply UP2 to find an upper bound. The other major parts of UP2_Z, including the lower bound of the optimal solution, the sub-gradient method and the solution process, are all the same as for UP1_Z. The idea behind UP2 is first to solve $pf^b_{i+1}$, $cfl^b_i$, and $pf^a_{i+1}$, then to solve $caff^b_i$ as well as $cf^b_i$, based on the obtained $cfl^b_i$. In particular, the UP2 steps in UP2_Z are the same as for UP1 in UP1_Z, except for steps 1–4, which are outlined below:
Step 1: Let the cargo flows for the initial lower bound solution be $c_f^a$.
Step 2: Solve $p_f^b$, $c_f^b$, and $p_f^b$ based on $c_f^a$ as follows. First, a modified combi-fleet-flow network (MCFFN) is constructed, the same as CFFN, except that the cost of each flight leg arc in MCFFN includes the original operating cost plus the sum of all profits obtained from the corresponding cargo delivery arc flows for $c_f^a$. Note that the sum of the profits is set to be at most the aircraft capacity. All other operating constraints, such as the fleet size, the available airport/airport pair quota, and the loading constraints, are the same as for Model (A). Then, use CPLEX to solve PFFN, MCFFN and PFNS together, to find $p_f^b$, $c_f^b$, and $p_f^b$.
Step 3: Solve $c_f^b$ and $c_f^b$ based on $c_f^a$. To do this, we fix the $c_f^a$ variables in CFFN and then, use CPLEX to solve CFFN, CAFFN and CFN together. Note that all the operating constraints are the same as in Model (A). We have found $c_f^a$ and $c_f^b$.
Step 4: By combining $p_f^b$, $c_f^b$, $p_f^b$, $c_f^b$, and $c_f^b$, we achieve an $f_{ubs}^1$, and its objective value, $ov_{f_{ubs}^1}$.

4.2.2. UP1_O and the UP2_O

To find a better initial lower bound, unlike UP1_Z/UP2_Z, we relax the integer constraint (17) in Model (A) and using CPLEX solve for the optimal solution of this linear program. We calculate the violated volume for every side constraint (i.e. constraints (5)–(13)), based on the linear optimal solution. Then, the initial Lagrangian multipliers in UP1_O/UP2_O are set to be the violated volume of the corresponding side constraints. All the major parts of UP1_Z/UP2_Z, including the lower bound of the optimal solution, the upper bound of the optimal solution, the sub-gradient method and the solution process, are all the same as for UP1_Z/UP2_Z.

It should be mentioned that the fleet flows obtained from the above process cannot yet be directly put into practice without identifying the route of each airplane. A flow decomposition method (Yan and Young, 1996) is applied to decompose the arc flows into arc chains. Each arc chain denotes an airplane’s route. Note that these arc chains may not be unique, while the objective values for different arc chain patterns are the same, since the fleet flows do not make any difference. In practice, carriers could apply several arc chain patterns provided they meet aircraft maintenance and crew scheduling constraints.

5. Numerical tests

To test how well the model and the solution algorithms may be applied in the real world, we performed numerical tests using operational data from a major Taiwan airline, with reasonable assumptions. The reasonable assumptions primarily mean that the data needed to estimate in the numerical tests, such as the average cargo fare rates, the average flight times, the average aircraft holding charges, and the average ground handling times, according to the airline's operating data and the government regulations. We used the C computer language, coupled with the mathematical programming solver, CPLEX 8.1, to develop all the necessary programs. The tests were performed on a Pentium 4 – 3.0 GHz with 2 Gb of RAM in the environment of Microsoft Windows 2000.

5.1. Test results

The numerical tests were mainly based on data obtained from a major Taiwan airline’s operations in Asia during 2002. There were 12 and 10 cities served by passenger and cargo services, respectively. Three types of aircraft were used, including five B767-300 passenger aircraft (226 seats each), 8 B747-400 combi aircraft (272 seats and 35 metric tons each), and 6 MD-11F cargo aircraft (80 metric tons each). The planning passenger and cargo load factors for each flight were set not to exceed 0.7 and 0.8, respectively. All the cost parameters and other fleet-flow, passenger-flow and cargo-flow time–space network inputs were set primarily based on the actual operating data and Taiwan government regulations. Altogether, the test contained three layers of fleet-flow time–space networks, 22 layers of passenger-flow time–space networks, and 28 layers of cargo-flow time–space networks, involving 82,243 nodes and 443,251 arcs (variables). The model, which was substantially large in terms of combinatorial optimization, included 158,246 constraints, in which 82,243 constraints were for
ensuring flow conservation and the other 76,003 constraints were side constraints. For ease of writing, we define the symbols used in the numerical tests as follows:

- **OFV**: the objective function value;
- **LBP**: a lower bound of the problem, which is the best objective value of all the unexplored nodes in the branch-and-bound tree obtained by using CPLEX;
- **$G$**: the gap between OFV and LBP.

To evaluate the feasibility of applying the exact solution method to the Model (A), we used CPLEX to solve Model (A) for different limited computational times. The results are shown in Table 1. We found that the $G$ was about 27.08% when the computational time was limited to 12 h. Even with a computational time of up to 72 h, the $G$ was about 15.01%. These results show that it is difficult to optimally solve the model simply by using a commercial optimization solver, such as CPLEX.

For ease of comparison, the OFVs were obtained by the four heuristic methods within a limited computational time of 12 h. The percentage of improvement (PI) obtained using CPLEX is defined as follows:

$$\text{PI} = \left( \frac{\text{OBJ of UP1}_Z/\text{UP2}_Z/\text{UP1}_O/\text{UP2}_O - \text{OBJ of CPLEX}}{\text{OBJ of CPLEX}} \right) \times 100\%$$ (18)

As shown in Table 2, in general, **UP2_O/UP2_Z** could provide a better result than **UP1_O/UP1_Z**. However, the OFVs obtained by the 4 heuristic methods were all better than those obtained by CPLEX. In particular, for **UP2_Z/UP2_O**, the PI was about 26.71/27.53%. Moreover, the $G$s of the four heuristic methods were all better than those of CPLEX. In particular, for **UP2_Z/UP2_O**, compared with CPLEX, the $G$ was improved from 27.08% to 7.61/7.01%. These results indicate that our heuristics are significant improvement over CPLEX, and have the potential to solve huge scheduling problems. Moreover, we also found that in **UP2_O** (with the best OFV), eight airplanes were used to provide 272 flights/week. The average passenger and cargo load factors in **UP2_O** were 66.76% and 69.82%, respectively. The passenger and cargo service rates of **UP2_O** were both the highest, 97.23% and 95.81%, respectively. These results show that **UP2_O** could use the resources more efficiently than could the other heuristics.

It should be mentioned that in **UP1/UP2** we have also tried different approaches to solve the problem. In particular, we randomly generated the cost of each flight leg arc, instead of using that obtained in Step 1 of **UP1**, and then solved MFFNs. This process was repeated to find the various $f^b_1$s. Then, we selected the best $f^b_1$ to be the feasible fleet flow in Step1 of **UP1**. In addition, compared with **UP2**, we also tried to solve $cff^b_1$, $caff^b_1$, and $cfow^b_1$ first, then to solve $pf^b_1$ as well as $pflow^b_1$, based on the obtained $cfl^b_1$. However, the obtained results were all inferior to that obtained by **UP2_Z/UP2_O**. That is, the $G$s of these approaches were all greater than 7.0%. These results indicate that it is practically difficult to optimally solve such a huge scheduling problem, with more than 443,000 variables and 158,000 constraints. The heuristics, **UP2_Z** and **UP2_O**, have the potential to be useful for the efficient solution of such problems.

### 5.2. Sensitivity analyses

To understand the influence of the parameters on the solution, we performed a sensitivity analysis of the passenger ticket and cargo fares as well as the passenger and cargo demands, all of which are essential inputs to the model. Note that sensitivity analyses of other factors can be similarly performed, but this is left for future research. For simplicity, we used the better heuristic, that is, **UP2_O**, for the analyses.

<table>
<thead>
<tr>
<th>Computational time (h)</th>
<th>OFV (NT$)</th>
<th>LBP (NT$)</th>
<th>$G$ (%)</th>
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<tr>
<td>12</td>
<td>-61,395,428</td>
<td>-84,195,584</td>
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<td>24</td>
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<td>72</td>
<td>-70,458,568</td>
<td>-82,897,575</td>
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5.2.1. Passenger ticket and cargo fares

The passenger ticket and cargo fares both directly affected carrier revenue. Here, we performed three fare sensitivity analyses, using a ±20% difference in the passenger ticket fare, the cargo fare, and both the passenger ticket and the cargo fares. As shown in Fig. 5, when the passenger ticket fares were increased to 110/120%, the OFV showed an improvement of 17.08/32.04%. A 1% passenger ticket fare increase led to a 1.6% increase in the OFV, which was, on average, equal to NT$1,252,738. When only the passenger ticket fares were decreased to 80/90%, the OFV was improved by 33.75/17.7%. A 1% decrease in the passenger ticket fare would cut 1.73% from the OFV, about NT$1,354,523 on average. Similar results were also found when the cargo fare,

### Table 2
Test results

<table>
<thead>
<tr>
<th></th>
<th>CPLEX</th>
<th>UP1_Z</th>
<th>UP1_O</th>
<th>UP2_Z</th>
<th>UP2_O</th>
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<td>G (%)</td>
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<td>13</td>
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<td># Aircraft used</td>
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<td>68.36</td>
<td>67.13</td>
<td>70.04</td>
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*Note: * not available.

Fig. 5. Sensitive analyses of the passenger ticket and the cargo fares.
as well as both the passenger ticket and the cargo fares, were changed. In particular, a 1% increase/decrease in
the cargo fare led to a 0.9% increase/decrease in the OFV, which was about NT$704,665 on average. As well, a
1% increase/decrease in both the passenger ticket and the cargo fares, led to a 2.57% increase/2.52% decrease
in the OFV, which was, on average, equal to NT$2,014,614/NT$1,975,265. In addition, we also found that the
passenger ticket fare had more influence on the OFVs than did the cargo fare, probably because the passenger
ticket fare’s elasticity was relatively high compared with that of the cargo fare.

5.2.2. Passenger and cargo demands

To evaluate the potential influences of changes in the passenger or cargo demands on the solution, three
demand sensitivity analyses, using a ±20% difference in the passenger demand, the cargo demand, and both
the passenger and the cargo demands, were performed. As shown in Fig. 6, when the passenger demand was
increased to 110/120%, the OFV was improved by 3.04/5.68%. When the passenger demand was decreased to
80/90%, the OFV was improved by 24.39/11.68%. Generally, a 1% increase/decrease in passenger demand
would lead to an increase/decrease of 0.29/1.19% from the OFV, which was, on average, about NT$1,252,738/NT$934,786
Similar results were also found when the cargo demand as well as both the pas-
senger and the cargo demands were changed. In particular, the OFV produced a 0.34% increase/0.43%
decrease, about NT$265,654/NT$340,368 on average, when there was a 1% increase/decrease in the cargo
demand. The OFV showed a 0.68% increase/1.64% decrease, about NT$531,476/NT$1,283,276 on average,
when there was a 1% increase/decrease in both the passenger and the cargo demands. Moreover, compared
with the passenger and the cargo demands, we found that the OFV, when there was a 110/120% passenger
demand, was similar to that for a 110/120% cargo demand. However, the OFV for a 80/90% passenger
demand was inferior to that for a 80/90% cargo demand, meaning that a decrease in the passenger demand
would yield a greater decrement of the OFV than would a decrease in the cargo demand.

6. Conclusions

In this research, on the basis of the carrier’s perspective, we develop an integrated scheduling model that
combines passenger, cargo and combi flight schedules. The model is capable of directly managing passenger,
cargo and combi flight source interrelationships and is expected to be a useful planning tool by which carriers
can determine suitable fleet routes and timetables for short-term operations. A network flow technique is
applied to construct the model. It includes multiple fleet-flow, passenger-flow, and cargo-flow networks. It
is formulated as an integer multiple commodity network flow problem that is characterized as NP-hard. A
family of heuristics, based on Lagrangian relaxation, a sub-gradient method, four self-developed upper bound
heuristics and a flow decomposition algorithm, is developed to efficiently solve the problem.
Numerical tests, utilizing data from a major Taiwan airline’s operations, were performed to evaluate the model and the heuristics. The OFVs obtained by the four heuristic methods were all better than those obtained by CPLEX, especially for a huge scheduling problem, which occur in practice with more than 443,000 variables and 158,000 constraints and is difficult to solve optimally. In particular, compared with CPLEX, the UP2-O yielded the PI of 27.53% and improved the G from 27.08% to 7.01%. These results show that these solution methods have the potential to be useful for solving such huge scheduling problems. Several sensitivity analyses were also performed, to help us to understand the influence of the essential parameters on the solution.

The contributions of the paper to the literature are to provide the integrated flight scheduling model and a family of heuristics. In particular, the developments of the integrated model and the solution algorithm for the integrated flight scheduling problem have not been discussed in the past research. In addition, the practical values of the integrated model have several points: (1) direct and systematic integration of flight sources, (2) effective management of passenger–cargo relationships, (3) speeding up scheduling process and enhancing cooperation among different processes, and (4) providing a systematic computerized tool, all of which are helpful for airlines’ operations.

Finally, the model and the solution algorithms proposed in this research should be useful as reference material for carriers to develop different models and explore different solution algorithms suitable for their own short-term operations. For example, other operating constraints (e.g., airport selection, maintenance and crew scheduling) or other objectives may be incorporated into the model. Furthermore, for practical large-scale problems, the heuristic method could be further improved. For example, the upper bound heuristic or the sub-gradient method may be improved. As well, other useful algorithms or modern meta-heuristic techniques, for example, column generation, genetic algorithms, tabu search methods or threshold accepting methods, may be developed or incorporated into the algorithm to help solve the problem. All of these model and solution algorithm developments could be a direction for future research.

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Appendix A. Models (B), (C), (D), (E), (F), and (G)

Using the previous defined notations and symbols, the formulations of Models (B), (C), (D), (E), (F), and (G) are given by

Model (B):

$$\text{Min} \sum_{m \in M} \sum_{i \in A_m} C_i^m X_i^m + \sum_{i \in L} \sum_{j \in D_i} O_{ij} P_i^l + \sum_{n \in N} \sum_{j \in B_n} T_{ij} Y_n^m + \sum_{n \in N} \sum_{j \in B_n} Y_{ij} (V_i + V_j)$$

$$+ \sum_{m \in M} \mu S \left( \sum_{i \in C} I_{ij}^m - A F_m \right) + \sum_{i \in F} \sum_{j \in F} X_{ij}^m - A F_m \right) + \sum_{i \in F} \sum_{j \in F} T_{ij} Y_n^m + \sum_{i \in F} \sum_{j \in F} Y_{ij} (V_i + V_j)$$

subject to constraints (2)–(4), (14)–(16), and (17).
Model (C):

\[
\min \sum_{ij \in A_1} C_{ij}^1 x_{ij}^1 + \mu_1 \left( \sum_{ij \in CF_1} X_{ij}^1 - AF_{ij} \right) + \sum_{ij \in FF_1 \setminus FF_2} \mu_2 x_{ij}^1 + \sum_{ab \in SA} \mu_3 x_{ij}^1 \]

subject to constraints (2), (14), and (17).

Model (D):

\[
\min \sum_{ij \in A_2} C_{ij}^2 x_{ij}^2 + \mu_2 \sum_{ij \in CF_2} X_{ij}^2 + \sum_{ij \in FF_3} \mu_3 x_{ij}^2 + \sum_{ab \in SA} \mu_4 x_{ij}^2 \]

subject to constraints (2), (14), and (17).

Model (E):

\[
\min \sum_{ij \in A_3} C_{ij}^3 x_{ij}^3 + \mu_3 \sum_{ij \in CF_3} X_{ij}^3 + \sum_{ij \in FF_3} \mu_4 x_{ij}^3 + \sum_{ab \in SB} \mu_5 x_{ij}^3 \]

subject to constraints (2), (14), and (17).

Model (F):

\[
\min \sum_{i \in L} \sum_{j \in D_i} Q_{ij}^i p_{ij}^i + \sum_{i \in FF_2} \mu_2 x_{ij}^i \]

subject to constraints (4) and (16).

Model (G):

\[
\min Z = \sum_{n \in N} \sum_{ij \in B_n} T_{ij}^n y_{ij}^n + \sum_{n \in N} \sum_{ij \in BF_n} Y_{ij}^n (V_i + V_j) + \sum_{ij \in FF_3} \mu_3 \sum_{n \in N} y_{ij}^n \]

subject to constraints (3) and (15).

References


