Stochastic Models of Telecommunication Systems

Assignment No. 1

Maximum marks: 40

1. A particle starts at the origin and moves to and fro on a straight line. At any move it jumps either 1 unit to the right or 1 unit to the left each with probability 1/3 or 1/3 respectively. With probability 1/3, the particle may stay at the same position in any move. Model this process as a stochastic process. Write the stochastic process with state space and parameter space. Verify whether this process satisfies Markov property.

   (1 + 2 marks)

2. A digital camera needs three batteries to run. You buy a pack of 6 batteries, install three of these batteries into the camera. Whenever a battery is drained, you immediately replace the drained battery with the one new battery from the available stock. Assume that each battery lasts for an amount of time that is exponentially distributed with mean $1/\mu$, independent of all other batteries. Eventually camera stops running, only two batteries will be left out in the camera that are not drained. Let $X(t)$ denote the number of batteries not drained at time t. Draw the state transition diagram for the process $\{X(t), t \geq 0\}$. Find the expected time that your camera will be able to run with the pack of batteries bought. (2 + 2 marks)

3. Consider New Delhi International Airport. Suppose that, it has three runway. Airplanes have been found to arrive at the rate of 20 per hour. It is estimated that each landing takes 3 minutes. Assume that a Poisson process for arrivals and an exponential distribution for landing times. Without loss of generality, assume that the system is modeled as a birth and death process. What is the steady state probability that the no waiting time to land? What is the expected number of airplanes waiting to land? Find the expected waiting time to land? (2 + 2 + 2 marks)

4. We consider a buffer that receives messages to be sent. The transmission is made by means of two modern lines that operate at the same speed. We know that:

   (a) The message arrival process is Poisson with mean rate λ

   (b) The message transmission time is exponentially distributed with mean value $E[X]$

   It is requested to determine the following quantities

   (i) The traffic intensity in Erlangs that is offered to the buffer,

   (ii) the mean number of messages in the buffer,

   (iii) The mean delay for a message from its arrival to the buffer till it is completely transmitted

   (iv) Could the buffer support an input traffic characterized by $\lambda = 10\ msg/s$ and $E[X] = 2\ s$?

   (2 + 2 + 2 +2 marks)

5. Consider the telephone switch system with finite number of subscribers 100. Each subscriber can attempt a call from time 0. Assume that 20 channels in the telephone switch system to handle the calls and assume that each call needs one channel. Assume that the arrival of call from each customer follows Poisson process with rate λ and each call duration follows independent exponential distribution with rate μ. Draw the stochastic Petri net model for this queueing system. Find the probability that all 20 channels are busy using reward rates. (4 + 1 marks)

6. A radio link adopts four equivalent parallel transmitters for redundancy reasons. The operational characteristics of the transmitters require that each of them be switched off (for maintenance or recovery actions) according to the Poisson Process with a mean interarrival time of 1 month. The technician that performs maintenance and recovery actions requires a time exponentially distributed with mean duration of 12 hours in order to fix the problem. We consider that two technicians are available. The exercise requires:

   (i) To define a suitable model for the system;

(ii) To determine the probability distribution of the number of down transmitters at a generic instant;

(iii) To express the probability that no transmitter is operational on this radio link.

(2 + 2 + 2 marks)

7. State True(T) or False(F) with valid reasons for the following statements. No marks for TRUE or FALSE. Without valid reasons, marks will NOT be given.

   (a) The mean waiting time in the $M/M/5/5$ system with mean arrival rate 2 per minute and mean service rate is 4 per minute is 0.5.
   Statement is .... The reason is that ....

   (1 mark)

   (b) Suppose the transmission time of a packet is assumed to be exponential distribution with parameter 2 per second. Given that, the packet is not yet transmitted in 3 seconds, the total expected transmission time of the packet is 5 seconds.
   Statement is .... The reason is that ....

   (1 mark)

   (c) Assume that emails arrive in a Poisson process at an average rate of 100 per hour. Also assume that the time taken (in seconds) for each transmission by a server follows a uniform distribution with parameters 20 and 30. The probability that the system is not empty in the long run is 25/36.
   Statement is .... The reason is that ....

   (1 mark)

8. Consider the central library of IIT Delhi where there are 4 terminals. These terminals can be used to obtain information about the available literature in the library. If all terminals are occupied when someone wants information, then that person will not wait but leave immediately (to look for the required information somewhere else). A user session on a terminal takes exponential distributed time with average 2.5 minutes. Since the number of potential users is large, it is reasonable to assume that users arrive according to a Poisson process. On average 25 users arrive per hour.

   (a) Determine the probability that i terminals are occupied, $i = 0, 1, 2, 3, 4$.

   (b) What is the fraction of arriving users finding all terminals occupied?

   (c) How many terminals are required such that at most 5% of the arriving users find all terminals occupied?

   (2 + 1 + 2 marks)

Deadline: Submission by email to me (dharmarajas@gmail.com) on or before June 19th 5:00 PM.